

THE CALGARY MATHEMATICAL ASSOCIATION

37<sup>th</sup> JUNIOR HIGH SCHOOL MATHEMATICS CONTEST

April 24, 2013

NAME: \_\_\_\_\_  
PLEASE PRINT (First name Last name)

GENDER:  M  F

SCHOOL: \_\_\_\_\_

GRADE: \_\_\_\_\_  
(9,8,7,...)

- You have 90 minutes for the examination. The test has two parts: PART A — short answer; and PART B — long answer. The exam has 9 pages including this one.
- Each correct answer to PART A will score 5 points. You must put the answer in the space provided. No part marks are given.
- Each problem in PART B carries 9 points. You should show all your work. Some credit for each problem is based on the clarity and completeness of your answer. You should make it clear why the answer is correct. PART A has a total possible score of 45 points. PART B has a total possible score of 54 points.
- You are permitted the use of rough paper. Geometry instruments are not necessary. References including mathematical tables and formula sheets are **not** permitted. Simple calculators without programming or graphic capabilities **are** allowed. Diagrams are not drawn to scale. They are intended as visual hints only.
- When the teacher tells you to start work you should read all the problems and select those you have the best chance to do first. You should answer as many problems as possible, but you may not have time to answer all the problems.

MARKERS' USE ONLY	
PART A _____ × 5	
B1	
B2	
B3	
B4	
B5	
B6	
TOTAL (max: 99)	

**BE SURE TO MARK YOUR NAME AND SCHOOL  
AT THE TOP OF THIS PAGE.**

**THE EXAM HAS 9 PAGES INCLUDING THIS COVER PAGE.**

**Please return the entire exam to your supervising teacher  
at the end of 90 minutes.**

**PART A: SHORT ANSWER QUESTIONS** (Place answers in the boxes provided)

A1 From the set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ , all odd numbers are removed. How many numbers are remaining?

A1

A2 A bag contains red, blue and green marbles.  $\frac{2}{3}$  of the marbles are not red and  $\frac{3}{4}$  of the marbles are not blue. What **fraction** of the marbles are not green? Express your fraction in **lowest terms**.

A2

A3 Ajooni walked 9 km at 4 km per hour, then biked for 4 hours at 9 km per hour. What was her average speed (in km per hour) for the entire trip?

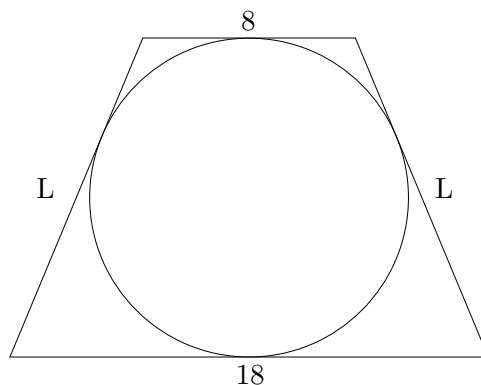
A3

A4 Notice that the digits of 2013 are four consecutive integers (because 0, 1, 2, 3 are consecutive integers). What was the last year (before 2013) whose digits were four consecutive integers?

A4

A5 A circle is inscribed in an isosceles trapezoid, as shown, with parallel edges of lengths 8 and 18 cm and sloping edges of length  $L$  cm each. What is  $L$ ?

A5



A6 Mary has a large box of candies. If she gives a third of her candies to her mom, then a third of the remaining candies to her dad, and finally a third of what's left to her little sister, there will only be 16 candies in the box. How many candies are in the box at the beginning?

A6

A7 I have half a litre of solution, which is 40% acid, and the rest water. If I mix it with 2 litres of solution which is only 10% acid, what is the **percentage** of acid in the mixture?

A7

A8 A two-digit positive integer is said to be **doubly-divisible** if its two digits are different and non-zero, and it is exactly divisible by each of its two digits. For example, 12 is doubly-divisible since it is divisible by 1 and 2, whereas 99 is not doubly-divisible, since its digits are equal, and 90 is not doubly-divisible, because it contains a zero. What is the **largest** doubly-divisible positive integer?

A8

A9 What is the remainder when  $2^{2013}$  is divided by 7?

A9

## PART B: LONG ANSWER QUESTIONS

B1 You currently have \$100 and two magic wands  $A$  and  $B$ . Wand  $A$  increases the amount of money you have by 30% and wand  $B$  adds \$50 to the amount of money you have. You may use each wand exactly once, one after the other. In which order should you use the wands to maximize the amount of money you have? How much money would you have?

B2 Put one of the integers  $1, 2, \dots, 13$  into each of the boxes, so that twelve of these numbers are used once (and one number is not used at all), and so that all four equations are true. Be sure to explain how you found your answers.

$$\square + \square = \square$$

$$\square - \square = \square$$

$$\square \times \square = \square$$

$$\square \div \square = \square$$

B3 On planet X, an X-monkey has 2 legs and one head, while an X-hypercow has 3 legs and 4 heads. Robert has a herd of X-monkeys and X-hypercows on his farm, with a total of 87 legs and 86 heads in his herd. How many animals of each kind does Robert have?

B4 A pie is cut into  $a$  equal parts. Then one of these parts is cut into  $b$  smaller equal parts. Finally, one of the smaller parts is cut into  $c$  smallest equal parts. One of the original parts, together with a smaller part and a smallest part, makes up exactly three fifths of the pie. What are  $a$ ,  $b$  and  $c$  (assuming  $a$ ,  $b$  and  $c$  are integers greater than 1)?

B5 In a hockey tournament, five teams participated where each team played against each other team exactly once. A team receives 2 points for a win, 1 point for a tie and 0 points for a loss. At the end of the tournament the results showed that no two teams received the same total points, and the order of the teams (from highest point total to lowest point total) was  $A, B, C, D, E$ . Team  $B$  was the only team that did not lose any games and team  $E$  was the only team that did not win any games. How many points did each team receive and what was the result of each game?

	Total Points
$A$	
$B$	
$C$	
$D$	
$E$	

	Winner (or tie)
$A$ vs $B$	
$A$ vs $C$	
$A$ vs $D$	
$A$ vs $E$	
$B$ vs $C$	
$B$ vs $D$	
$B$ vs $E$	
$C$ vs $D$	
$C$ vs $E$	
$D$ vs $E$	



B6 The three edges of the base of a triangular pyramid (tetrahedron) each have length 6 units, and the height of the pyramid is 10. The other three (sloping) edges are equal in length. A sphere passes through all four corners of the pyramid. What is the radius of the sphere?

